
A taste of engineering mathematics from present-day Russia

1. Preface

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It was not a simple task, but we did it! The *Russian Issue* is in your hands. A goal of the Issue Project, as it was formulated about three years ago, was to present to the English-speaking community in the field of Engineering Mathematics a valuable level of research which is under way in modern Russia. It was assumed from the very beginning that the international community is not quite familiar with Russian researchers and their work. This simplified approach was definitely incorrect and the editors tried to specify the goal more precisely. Many Russian scientists are already well “incorporated” into the international scientific community; they publish their papers in international journals and take part in international projects with partners from other countries. Researchers of Russian origin, who permanently or mainly live abroad, were not contacted with a request to contribute to this project. The idea behind the project was that only Russian researchers who permanently work in Russia may provide “a taste from Russia”. Moreover, it was not allowed to submit papers that were written in collaboration with colleagues from other countries, including researchers from Ukraine. The latter was not easy because Russian and Ukrainian researchers had very strong connections in the past; many of them still remember the time when they represented just one scientific community.

Many people throughout Russia, who are working in the field of applied mathematics, were asked to contribute. Not all of them promised to contribute. A reason for this might be the dangerously high intensity of scientific work, not only in Russia, but in any country. Research or scientific projects which give no immediate profit often have to be dropped in favor of projects (sometimes many of them) which should be finished in a short time. Researchers are forced to do many things at the same time, including administrative duties which are sometimes considered as more important than research itself. Some researchers who were asked to contribute at the start of this project replied that they were too busy and their time had already been scheduled for years ahead, so they could not contribute. We did not force such colleagues to contribute since we knew that their reasons for not contributing were genuine.

Originally, three Guest-Editors were working on this Issue: two from the European part of Russia and one from the Asian (or Siberian) part. These three editors recruited authors for the issue. Later, one of the editors, Professor Indejzev, was appointed as the Director of his

Institute in Saint Petersburg. He informed the others that he could not continue his duties as editor of the issue. Professor Erofeyev did a lot in the way of inviting potential authors to contribute. When the papers started arriving at the Editors' (now only two of them), another important part of the editorial work began, namely to find suitable refereeing experts, communicating with them and the authors, dealing with the revisions and, finally, acceptance or rejection as the case might be. We did not promise the authors beforehand that their papers would be accepted and published whatever. The contributors were informed that their papers would be peer-reviewed as any other regular submission to the Journal of Engineering Mathematics. For some authors this was a rare experience because, so far, none of their papers had been subjected to a thorough review process that involved experts from other countries. Moreover, some contributors to this issue for the first time went through all the stages of submission, reviewing and revising their papers for a truly international journal. It was important that such authors be carefully and gently guided by the editors, who explained to them what the referee had in mind and how to proceed with the reviewing of their manuscripts. Sometimes the editors and some of the referees actually edited a paper with the aim of improving its presentation and turning it into an acceptable form. Needless to say that, because of the different standards of presentation, the reviewing processes sometimes proceeded far from smoothly. I was already familiar with this work, which is why I took it upon me together with associated responsibilities. However, some of the submitted papers were quite far from my own field, which brought certain difficulties to the reviewing process.

Our thanks go to the many referees from many countries for studying the papers, providing their comments and offering valuable suggestions. Some referees actually edited papers assigned to them, sometimes returning heavily marked manuscripts. Some of them did heroic work trying not only to review a paper, but also wanting to understand what the authors had to say and what was the gist of their papers. The substandard quality of the English used in the original submissions was a source of irritation with some of them. This is why we tried to separate their strictly scientific observations from those concerning linguistic matters, the terminology used and the presentation. This was done intentionally. For this issue we tried to keep the accepted papers as close to the original versions as possible. We believe that, had we not done this, the papers in this issue would have been indistinguishable from regular JEM papers and "A Taste of Research in Modern Russia" would not be imparted. Eventually some adjustments had to be made without sacrificing the Russian flavor. In the final stage of the project, the Editor-in-Chief, Professor Hendrik K. Kuiken, decided to screen all the papers and bring these to a level of presentation and English usage that is deemed acceptable for an international journal.

The research community is international, which is good. They speak and write English and follow well-accepted formats of presentation and research itself, thus saving a lot of time. On the other hand, I hope the reader will have no difficulty with the idea that a variety of people working in a scientific area, each with their different scientific and cultural backgrounds, different "eyes" and different "brains", is an important factor for successful research. Admittedly, "unified" backgrounds and "unified" researches render communication, collaboration and other things much easier, but also, somehow, create a "distance" towards the final result and may hamper finding a final answer. This will not always be the case, but the possibility of this happening should be taken into account.

Presenting their original results in English was not a simple task for some contributors to this issue. For those doing fundamental research it is rather natural to communicate and collaborate with scientists from other countries. On the other hand, researchers involved in applied and certainly in engineering mathematics, which usually is closely related to their

national industry, are used to presenting their work in their native language. Their interest and allegiance are tied up with their national companies rather than with the international scientific community. Nowadays, everybody should know English, if only to follow modern trends, and understand new technologies and techniques. However, to understand technical papers in English and to present your own results in English are two different stories. In the past, when Russia was part of the Great Soviet Union, there was no real need to know even technical English, despite the fact that all Russian researchers studied English (and German and French, for that matter) for many years in school, the university and finally as part of PhD courses. A reason was that, in the past, most important papers published in English (or other languages) were urgently translated into Russian. These translations were of very high quality; many of these were edited by top-level scientists who added deep comments of their own, corrected misprints (sometimes mistakes), added more references, explained obscure parts of the paper and discussed possible future developments of the research topic discussed in the paper. It is my private opinion, as is everything else in this Preface, that sometimes reading these translated papers together with the deeply informative comments of the translating editor, I have the feeling that those comments are at least as important as the paper itself. On the other hand, even if your English is not good enough, you always had the possibility of publishing your paper in a good Russian journal, which is translated into English cover-to-cover. Nowadays the English version of your paper appears almost at the same time as the Russian original. However, in this case a Russian paper is reviewed by Russian referees and the translated version could be rather formal. Moreover, such journals still are not as widely available as, for instance, the *Journal of Engineering Mathematics*.

The contributors to this issue come from different fields and have different levels of experience; their papers are devoted to a wide range of topics. This is not an issue on just a single subject. However, we believe that specialists will find new ideas, new formulations, helpful techniques and outstanding results in these papers.

The issue opens with the paper by Saren *et al.* which investigates the hydrodynamic interaction of airfoil cascades, the one moving with respect to the other. A semi-empirical theory of potential-vortical interaction of two cascades is employed for various gaps between the cascades and their pitches. The theory was also applied to the problem of three cascades of stator-rotor-stator type with the aim of describing the so-called clocking effect. The vortical interaction of cascades is described within a semi-empirical model of turbulent diffusion. To validate the theoretical models, the obtained results are compared both with numerical and experimental ones.

In the paper by Paryshev you may find approximate solutions of problems in high-speed hydrodynamics concerning the dynamics and stability of a slender ventilated cavity. Further topics that are dealt with are nonlinear oscillations of the cavity and its closure and the impact of an expanding cylinder onto the curved free surface of a liquid. The author stresses the point that, despite the tremendous progress in computational fluid dynamics, methods based on relatively simple physical considerations, as well as approximate mathematical models, are still of practical importance. This is confirmed by a comparison of analytical results obtained by the author with experimental data.

Afanasiev and Grigorieva investigate numerically three-dimensional bubble dynamics in the presence of rigid boundaries. A numerical algorithm based on a boundary-integral formulation is described in detail. The authors are concerned with the final stage, when the bubble collapses with the formation of a jet. They studied the jet length, its speed and the direction of its motion depending on the initial bubble position and its dimension. Such problems are difficult to handle, both numerically and analytically, because the flow topology changes when

the jet starts to penetrate the free surface of the bubble. In hydrodynamics, problems with variable flow topology are still not well studied.

A model of a double continuum with variable local topology is put forward in the paper by Aero. This model is able to describe some inelastic phenomena in continuous media, such as failure processes and plastic deformation of the media. Two kinds of atoms are introduced. The lattice of a continuum is composed of two periodic sublattices, each consisting of atoms of a single kind. The interaction between the sublattices is described by means of a nonlinear periodic force, which depends only on the displacements of the atoms. Equations governing double-continuum dynamics are derived and used to describe elastic and inelastic deformations with phase transitions and defect transformation.

The paper by Kulikovskii, Chugainova and Sveshnikova stresses the point that, for hyperbolic equations in continuum mechanics, non-uniqueness of their solutions is the rule rather than the exception. The authors considered one-dimensional self-similar problems for waves in an elastic half-space generated, in particular, by a sudden change in the boundary stress; this is the so-called “piston” problem. It is demonstrated that these problems, the solutions of which involve Riemann waves and evolutionary discontinuities, should be regarded as limiting cases of properly formulated regular problems within visco-elastic models of a medium, when the effects of viscosity and dispersion vanish or time tends to infinity.

Terentiev and Zhitnikov investigate steady free-surface flows with capillary effects taken into account. The unknown complex potential is given as a sum of three terms. Two of these are given analytically and serve to take the flow singularities into account; the third is regular and can be readily computed to a prescribed accuracy. It is shown that, for large Weber numbers, the free surface is covered with small-amplitude capillary waves.

The hydroelastic problem of wave impact onto a horizontal elastic plate is considered by Korobkin and Khabakhpasheva. During the initial stage of the interaction, the plate deflections are obtained by a modified modal method that does not require calculation of the hydrodynamic loads. The wetted part of the plate is unknown beforehand and is determined as part of the solution. The system of ordinary differential equations with respect to the principal coordinates of the plate modes is supplemented with nonlinear differential equations for the parameters of the wetted part of the plate. Different kinds of impact are studied, including plate-wave impact with attached cavity. The contributions of the kinetic and potential energies of the system to the total energy are analyzed. It is shown that the kinetic part of the energy of the system is small at the instant of time when the bending stresses in the beam approach their maximum values. This observation made it possible to estimate the maximum bending stresses in the plate.

Kovtunenکو studies constrained minimization problems in contact mechanics. In these problems, not only equations of motion and boundary conditions, but also one-sided inequalities must be satisfied on the boundary. The part of the boundary, along which an elastic body is in contact with a rigid obstacle, is referred to as the active set; the remainder of the boundary is the inactive set. By employing a shape-sensitivity analysis, the author obtains asymptotic expansions of the solution, as well as asymptotic expansions of some global problem characteristics with respect to small perturbations of the contact region between the elastic body and the rigid obstacle.

An asymptotic analysis of a two-dimensional linear problem for water waves generated by a moving external pressure was performed by Kuznetsov. He considers the problem of a pressure distribution traveling along the water surface and changing its velocity during a short time interval. A two-scale expansion of the velocity potential has been obtained, the small parameter being the non-dimensional duration of the acceleration stage. The asymptotic

behavior of the drag force was studied for different bottom topographies. The asymptotic expansions are rigorously justified.

A general analysis of linear problems with several scales in space and time is presented by Belov, Dobrokhotov and Tudorovskiy. These problems are studied by separating “fast” and “slow” modes and “freezing” the slowly varying variables. The approach developed in their paper is applied to some problems concerning internal waves in a pycnocline, water waves, wave propagation in nanofilms and wave dynamics in nanotubes, electron waves in crystals and electromagnetic waveguides and optics. The asymptotic solutions are rigorously justified.

Gladyshev, Gladysheva and Zubarev present theoretical and experimental results on new effects in the optics of moving media. An exact analytical solution was obtained for the trajectory of the wave vector of a monochromatic electromagnetic plane wave in a medium undergoing a complex motion. It is shown that spatial dragging of an electromagnetic wave by a moving medium can be correctly described when relativistic terms of higher order are taken into account. Results of a theoretical calculation of the expected interference-pattern shift are in agreement with available experimental data. The authors conclude that the revealed effects might be responsible for some observed phenomena, even if the velocities of the moving media are non-relativistic.

Unsteady electromechanical machining is considered by Zhitnikov, Fedorova, Sherykhalina and Urakov. They exploited the analogy of this problem with the famous Hele-Shaw problem. Exact solutions of some steady (in a moving coordinate system) and self-similar problems were derived. The obtained results show that, in the considered processes, a formation of stable configurations, close to self-similar or stationary ones, takes place sequentially or simultaneously. These configurations are described by the analytical solutions obtained in the paper.

Morozov and Nazolin study both deterministic and random processes in cyclic and dynamic systems. Some types of electric engines and generators are considered as cyclic dynamic systems with rotating components (shafts). Often the shaft rotation is non-uniform, which is why difficulties arise in the analysis of the corresponding problems. Random processes lead to fluctuations of the cycles of the motions of the system. These fluctuations are the subject of this study.

Balandin and Kogan deal with the difficult problem of robust design in the presence of structured uncertainties. The authors propose a technique for solving this linear-system-theory problem approximately. Meaningful bounds on the optimal values of the Lagrange-multiplier parameters were found, which is important for design purposes.

Chashechkin, Baydulov and Kistovich suggest that approximations to the original Navier–Stokes equations should be made so as to preserve as many symmetry properties as possible. Not only continuous symmetry groups, but also discrete symmetries are considered by the authors. An example of an application of the latter is given in terms of Rayleigh–Bénard convection. The paper continues with a discussion on periodic motions in fluids that are stratified by some physical factor and emphasizes that waves co-exist with small-scale boundary-layer phenomena. A classification of 3D periodic motions of stratified and rotating fluids is given with fluid compressibility and viscosity taken into account.

Inverse problems related to stratified flows are investigated by Anosova, Potetunko and Scherbak. They developed asymptotic and numerical algorithms by which one is able to determine fluid-density profiles by using the measured wave characteristics. Restrictions on the accuracy of the input data, which guarantee that the inverse problem can be solved to a prescribed accuracy, have been obtained. Finally the authors demonstrate that geometrical and physical characteristics and possible imperfections of elastic rods and elastic layers can be recovered by using the measured resonance frequencies of these elastic bodies.

The Russian Issue starts with two papers on turbo-machinery and cavitation which involve engineering mathematics to a high degree of sophistication. More details on cavity dynamics can be found in the third paper, which relies heavily on modern numerical methods. The following five papers are concerned with modern problems in elasticity, hydroelasticity, contact mechanics and with mathematical methods in elasticity. Two papers deal with the asymptotic analysis of complex problems in hydrodynamics and physics using multi-scale methods. The following four papers are devoted to problems in optics, electrochemical machining and mechanics. Finally, two papers are about symmetry properties of viscous and stratified flows.

We expect that this Special Issue will be helpful for specialists in Engineering Mathematics, promote the interest in papers by Russian scientists and give you a taste of Russian research.

2. The rationale behind this special issue

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For most of the last century there existed a semi-impermeable wall between the Soviet Union and the West, hampering the exchange of ideas and the mixing of scientific cultures. This has greatly affected the way engineering and applied mathematics developed in these two parts of the world. During the early part of this period of separation, the relationship between the two blocks was characterized by a kind of indifference and aloofness in matters scientific on the part of the West, particularly the U.S.A., *vis-à-vis* developments in the Soviet block. The language barrier played an important rôle in this. Whereas the Soviets made a point of translating much of Western science into Russian, this happened on a much smaller scale with Russian works into English. Notable exceptions are the series on Theoretical Physics by Landau and Lifshitz and translation programs by Princeton University Press, often cheaply produced on a type-writer, Noordhoff Publishers and some others. Veritable gems and inspiring works came to the West via these programs, *e.g.* Krylov and Bogoliubov's *Non-Linear Mechanics*, Muskhelishvili's inimitable works on mechanics, Mikhlin's *Integral Equations* and the like. For those in Europe and capable of reading German, East-German translations by VEB, Deutscher Verlag der Wissenschaften, Berlin, provided cheap, but profound editions of a range of Soviet books on mathematics. The five-volume *Lehrgang der höheren Mathematik* by V.I. Smirnov was used by many students in those days, giving them an excellent introduction into the subject of their choice.

However, it was not only the language barrier that kept the West less informed about scientific developments in the former Soviet Union. It was also a sense of superiority, again particularly on the part of the U.S.A., that made the West indifferent towards scientific developments in the Soviet Union. Whenever "they" made great strides forward, the accusation of espionage was easily levelled, rather than the admission that the other side might have reached a level of scientific and technological prowess comparable to that of the West. The most poignant example is, in this respect, the Soviets' atomic bomb that exploded only a few years after similar devices had been tested in the U.S.A. Supposedly, espionage was behind every major Soviet development in military technology, whether it be jet fighters, submarines, rockets, etc. No doubt, espionage has played an important rôle during all those years, and perhaps

more so on the one side than on the other. The implication, however, that “they” could never achieve such technologies on their own has, in retrospect, proved to be unsustainable.

The latter truth was brought home, with a vengeance, when Sputnik began sending its monotonic beeps to us from space and later when photographs were sent to Earth depicting the other side of the Moon. This sent shock waves through the West. All of a sudden it was clear that the Soviets were capable of doing research at the leading edge of science and technology unaided by others. It was admitted that these capabilities had been sorely played down and often entirely overlooked. An intensive program towards the translation of important Soviet science journals was set up. President Kennedy spoke his famous words about “this nation sending a man to the Moon and bringing him safely back to Earth before the end of the decade.” A highly speculative and seemingly unrealistic project at the time, but gloriously executed into what many regard as the greatest technological achievement of mankind. Clearly, these events would never have taken place, certainly not with the speed they did, but for the adrenalin shot provided by the sudden emergence of Soviet technology on the scene.

Arnold Toynbee, the famous British historian, writes [1, Chapter 1] that Russia has played the game of technology catch-up on a number of occasions. They felt compelled to do so when, during various stages of their history, they had been invaded by the Swedes, the Poles, the French, the Germans, etc. In this respect, it is worthwhile to recall that Czar Peter the Great travelled to Holland, deciding to work as an apprentice on a shipyard. Towards the end of the seventeenth century, the Dutch were regarded the foremost nation in shipbuilding, which is why he chose to go there to learn the tricks of that trade. Apart from the inevitable lackeys, he probably had many would-be engineers in his retinue. In the years that followed, shipyards, molded on what they had seen in Holland, sprang up on the shores of the Newa in Saint Petersburg, making sure that the Russian fleet would be equal to anyone else's. It should be mentioned that Czar Peter was greatly interested in mathematics and geometry, not only for their own sakes, but also with a view to applications in ship research and astronomy. In 1701 he founded the School of Mathematics and Navigation in Moscow. This serves to show that having someone interested in technology at the top may strongly influence the choices made for a country's development. With this in mind it should be noted that Hu Jintao, China's current leader, has a degree in hydraulic engineering.

A similar development took place in the twentieth century, but on a much grander scale. This is not the place to discuss the political motivations for this. What I am interested in is the effect the semi-impermeable wall, which had been erected between the then Soviet Union and the West, had on the separate development of the sciences and, for us, on engineering mathematics in particular. In my capacity as the E-i-C of this journal I read referee reports from time to time in which the referee complains about a lack of Western references appearing in a manuscript originating from the lands that formerly were part of the Soviet Union. What such referees, and many others with them, forget is that the reverse is equally true. In those earlier days, when most Westerners did not read the Russian literature in the raw, particularly reports, lines of research in applied/engineering mathematics were developed in the Soviet Union that the West left untouched until much later. Indeed, in later years discoveries were made, or rather duplicated, by Western workers in fields that their Soviet counterparts, with good reason, regarded as well-trodden. Nonlinear phenomena, applications of conformal-mapping techniques to intricate contexts in industrial mathematics, approximation methods, again applied with a view to solving practical problems, are examples of fields to which Soviet colleagues made great contributions long before Western workers got interested in these.

It should also be mentioned that Russian applied and engineering mathematics have retained a strong analytical foundation to this very day. Asymptotic methods and other ways

of approximating seemingly unmanageable models have always taken a central position within this analytical approach. One reason for this may be that technologies, particularly concerning the hardware enabling large-scale computing methods being implemented, were developed to a large extent by commercial enterprises outside the Soviet Union, particularly in the U.S.A. In the West virtually every university and research establishment made a point of acquiring the latest computer hardware, whereas in the Soviet Union these technologies were available mainly in big research establishments that were devoted to large projects. The geopolitical circumstances at the time were such that Western technology had difficulty seeping through the semi-impermeable wall. Hence, in the West, numerical techniques have gotten the upper hand over analytical approaches, whereas their Soviet colleagues had to devise clever analytical techniques to get their models to work for them in a practical sense. Now that the Soviet empire no longer exists, these differences in approach are disappearing. However, we still see a tendency for Russian and Ukrainian scientists to use their analytical capabilities to the full before embarking on actual numerical calculations.

I believe that this socio-political experiment, which has manifested itself over much of the twentieth century, and which has led to two quite separate developments in engineering and applied mathematics, is unique. It deserves to be studied by historians of science. We may never see a separate development like this again, what with the almost instant dissemination of knowledge afforded to us nowadays by the internet. On a smaller scale there is, of course, a wall separating military research from the outside world of science. This wall, however, does not cause a separate development of engineering mathematics, outside and inside. It simply prevents knowledge of gadgets and methodologies invented inside from getting out.

The situation that motivated the afore-mentioned separate development came to an end in 1990. As is true in thermodynamics, when a wall separating two systems is removed, diffusion sets in. Many scientists from the former Soviet Union left their country of birth and tried their luck in the West, particularly in the U.S.A. Even so, the majority stayed home, but many of them got the opportunity to travel and foster intensive connections with Western science. Later on, the internet facilitated this development enormously. Thus, it can be expected that the separate schools of thought will slowly lose their different characteristics. Clearly, many differences can still be seen. To this very day, much of what is published in Russian does not reach Western scientists. It can be expected though that the best work originating from Russia, Ukraine and other states that formerly made up the Soviet Union, will find its way to the international scientific press and be available in English right from the start. The completion of the diffusion process may be expected to occur within a few decades. Now this is exactly the motivation and rationale behind this special issue entitled *A Taste of Engineering Mathematics from Present-Day Russia*. The idea was to bring together in a single volume about fifteen contributions originating entirely from Russia proper. This way the likelihood of getting papers written and composed according to the old style was greatest. I realize full well that an issue like this, limited in scope as it is, can not claim to giving a complete overview of engineering mathematics as it is practised in the Russia of today. Hence the words "A Taste". I simply asked a very capable Russian applied mathematician, Professor A.A. Korobkin of Novosibirsk, to collect about fifteen papers from his particular network. My thanks are due to him for the considerable effort he invested into this project. He and I can only hope that this special issue will later be regarded an interesting document of the time.

Reference

1. A. Toynbee, *The World and the West*. London, New York, Toronto: Oxford University Press (1952) 99 pp.